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By the Design and Implementation of Modified Kalman Filter for LPV Systems

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***Abstract*—**Linear Parameter Varying (LPV) system is an important class of system, as it covers many physical systems. In this paper, the routine Kalman filtering scheme derivations are entertained to modify for generalized LPV systems. The original system is unstable, for controlling purpose a state-feedback controller is employed. For simulation purpose, a real time case study of Boeing-747 model is adopted. The results comprehend attractive features for modified Kalman filtering scheme.

***Keywords*—** Kalman Filter, Linear Parameter Varying, Linear Quardratic Regulator.

I. Introduction

State estimation is a vast area of research in control and communication system engineering. When the data signal overlaps with the noise signal, and ordinary filtering methods fail to work, state estimation comes into being. Perhaps the best estimator for the estimation problems of Linear time invariant systems is Kalman filtering [7]. However, it depends heavily on perfect knowledge of the system dynamics, information of unmeasured stochastic inputs and noisy measurement data [11]. Nonetheless, extraction of accurate system information from noisy sensed measurements is the fundamental goal of the Kalman filter [20].For this purpose the minimum mean square error estimate of the system state are calculated by the sensor measurement values. There are several estimation methods which are capable for linear, nonlinear and LPV systems. State estimation for LPV system is an attractive area of research due to it's wide applications.

LPV system is an emerging research area. It covers a wide range of systems, including UAVs, turbofan engines, missiles, which are most common applications of LPV systems. In some situations due to the presence of uncertainties (unknown parameters and/or faults) the design of a conventional estimators that converge in the noise-free fall to the exact value of the state, is complicated [6]. An efficient way to obtain LPV model through a collection of linear models obtained at various conditions [12], which will cover the variation of dynamics of a system. A common application area for LPV systems is aerospace problems. Recent work on estimation for LPV system is nonlinear approach and hence application of Extended Kalman filter (EKF) [13][14], but it has the complexity of nonlinear approach. In this paper the nonlinear approach is avoided and a routine Kalman filter is introduced, which gives remarkable results.

There is a considerable research work on LPV observers designs [19][9][2][4][16][10][8] and also on LPV control using various type of controllers [1][15][21]. Many results of stability using gain scheduling methods have been shown [17][5], in addition Kalman type realization of LPV system is used in [3]. The LPV model is taken from [12] and is regulated with a controller named, linear quadratic regulator (LQR). A numerical example of LPV system is taken from [1], for which LPV Kalman filter is implemented.

This research paper is ordered as follow. Section II describes the generalized form of LPV model. Section III presents LPV system with process and measurement noises. Section IV presents the basic derivation involved in LPV Kalman filter. Section V describes design of a standard state feedback controller. Section VI shows a case study of Boeing 747 series 100/200 and simulation results with and without employment of state feedback and LPV Kalman filter. Section VII conclude the paper with emphasis on main topic.

II. Linear Parameter Varying System Model

Linear parameter varying systems are basically linear systems whose dynamics vary linearly with a time varying parameter, say $\rho(k)$. The parameter itself is time varying but it has got specific bounds [12]. That is, $\rho(k) \in \varphi(k)$ where $\varphi(k)$ is a compact set. This varying parameter may be the state of the system. At lower bound of the parameter the system has got specific behavior while at the upper bound, the system has got another specific behavior. A generalized LPV system with varying parameter $\rho(k)$ can be represented as

$$\begin{bmatrix} \dot{x}(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A(\rho(k)) & B(\rho(k)) \\ C(\rho(k)) & D(\rho(k)) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \quad (1)$$

Where $A(\rho(k)) : R^{s} \to R^{n\times n}$, $B(\rho(k)) : R^{s} \to R^{n\times m}$, $C(\rho(k)) : R^{s} \to R^{p\times n}$, and $D(\rho(k)) : R^{s} \to R^{p\times m}$, are parameter dependent system dynamics, $x_k : R^{s} \to R^{n\times 1}$,

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$u_k : R^s \to R^{p\times1}$, $y_k : R^s \to R^{p\times1}$ represent state of a system, deterministic input and output of a system respectively. The elements $A(\rho(k)), B(\rho(k)), C(\rho(k))$ and $D(\rho(k))$ are polynomial functions of of a bounded parameter $\rho(k)$. Hence varying the parameter $\rho(k)$, the corresponding elements and the system matrices also vary. Variation in the system matrices causes variation in the system response.

III. INTERRUPTED SYSTEM MODEL

It is hard to imagine, a system free of interruption (noise, disturbance, etc) in practical scenario. Therefore, the interrupted version of the above mentioned LPV system, would have the following dynamics.

$$\begin{bmatrix} \dot{x}(k) \\ y(k) \end{bmatrix} == \begin{bmatrix} A(\rho(k)) & B(\rho(k)) \\ C(\rho(k)) & D(\rho(k)) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}$$

Where $w(k)k : R^s \to R^{n\times1}$ and $v(k)k : R^s \to R^{p\times1}$ are process and measurement noise vectors respectively. These noises are assumed to be uncorrelated, White Gaussian, zero mean and a bounded covariance matrices as $w(k) \approx \boldsymbol{N}(0, Q_k)$ and $v(k) \approx \boldsymbol{N}(0, R_k)$ where Q_k and R_k are covariance matrices. Noise process are uncorrelated with u_k and x_k having a joint covariance matrix given by

$$\mathrm{E}\begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \begin{bmatrix} w^T(s) & v^T(s) \end{bmatrix} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{ks} \qquad (2)$$

Where $Q \in R^{n\times n}, S \in R^{n\times p}$ and $R \in R^{p\times p}$.

IV. DISCRETE TIME LPV KALMAN FILTER

Discrete time Kalman filter for LTI systems have been shown in numerous literature such as [18][11]. In this section, the existing discrete time Kalman filter is modified for the above mentioned interrupted LPV system model. Kalman filter, a state estimator, predicts state ofthe system, using a previous output and input data samples. Consider a discrete time LPV system

$$x_{k+1} = A(\rho(k))x_k + B(\rho(k))u_k + w_k \qquad (3)$$

$$y_k = C(\rho(k))x_k + v_k \qquad (4)$$

Where $A(\rho(k)) : R^s \to R^{n\times n}$ is parameter dependent system transition matrix, $B(\rho(k)) : R^s \to R^{n\times m}$ is a parameter dependent input matrix, $C(\rho(k)) : R^s \to R^{p\times n}$ is parameter dependent output matrix. $x_k : R^s \to R^{n\times1}$, $u_k : R^s \to R^{p\times1}$, $y_k : R^s \to R^{p\times1}$ represent state of a system, deterministic input and output of a system respectively. $w_k : R^s \to R^{n\times1}$ is process noise and $v_k : R^s \to R^{p\times1}$ is measurement noise.

A. Prediction Step

In prediction step, state of a system is predicted as shown

$$x_{pred} = A(\rho(k))x_{up} + B(\rho(k))u_k \qquad (5)$$

The covariance matrix of the corresponding error is given by

$$P_{pred} = E[e_k e_k^T] \qquad (6)$$

$$P_{pred} = A(\rho(k))P_{up}[A(\rho(k))]^T + Q_d \qquad (7)$$

In equation (7), $[A(\rho(k))]^T$ is transpose of parameter depe-ndent transition matrix $A(\rho(k))$. The change in actual and predicted output known as innovation is given as

$$Inn = y_k - [C(\rho(k))]\, x_{pred} \qquad (8)$$

Where y_k is the actual output and $[C(\rho(k))]\, x_{pred}$ is the estimated output.

B. LPV Kalman filter Gain

The optimal value of Kalman filter gain matrix can be calculated as

$$K(\rho(k)) = P_{pred}\,[C(\rho(k))][S(\rho(k))]^{-1} \qquad (9)$$

Where

$$S(\rho(k)) = [C(\rho(k))]P_{pred}\,[C(\rho(k))]^T + R_d \qquad (10)$$

It can be seen, as expected that Kalman filter gain elements depend on linear parameter varying elements depends on sampling period T_S.

C. Update Step

The updated state estimation and hence the corresponding updated error covariance can be found as

$$x_{up} = x_{pred} + [K(\rho(k))]\, Inn \qquad (11)$$

Where $K(\rho(k))$ is Kalman filter gain matrix. Update error covariance is

$$P_{up} = (I_4 - [K(\rho(k))][C(\rho(k))])\, P_{pred} \qquad (12)$$

Equation (6-12) describe the basic steps involved in Kalman filtering. Kalman filter theory implies that the innovation term is uncorrelated with x_k and u_k ,

$$E[x_k Inn^T] = 0_{n\times p} \qquad (13)$$

$$E[u_k Inn^T] = 0_{(n+sm)\times p} \qquad (14)$$

Parameter $\rho(k)$ is also uncorrelated with x_k . For an affine LPV system, the use exhibit the following properties;

$$E[x_k \rho(k)^T] = 0_{n\times sn} \qquad (15)$$

$$E[u_k \rho(k)^T] = 0_{n\times sm} \qquad (16)$$

On the other hand, $E[x_k v_k^T] = 0_{n\times p}$, which results in

$$E[\hat{x}_k x_k^T]C^T = 0_{n\times p} \qquad (17)$$

It shows that either $E[\hat{x}_k x_k^T]$ is rank deficient matrix with its rows lying in the orthogonal complement of C. or $\hat{x}_k$ and x_k are uncorrelated. For cross covariance matrix to be rank deficient $\hat{x}_k$ and x_k must be dependent vectors, which is not possible. As a result $\hat{x}_k$ and x_k are uncorrelated.

V. State Feedback Controller Design

For controlling purpose a state feedback controller namely Linear Quadratic Regulator (LQR) is employed in this paper. The state feedback gain elements are arranged to achieve very fast eigenvalues (considerably faster than the average rate of change of operating point with a known upper bound) by solving the Algebraic Riccati Equation (ARE) with suitable weighting matrices. Given uniform observability and uniform controllability of key matrix pairs, the observer gain is used to stabilize the plant, even in the presence of fast (though not arbitrarily fast) variations in operating point by using the solution of a time-varying Riccati Differential Equation (RDE) [5]. The main advantage of LQR controller is that it can be used for time varying system dynamics. LQR is used for controller setting that minimize the undesired deviations and algorithm for minimizing the cost function, where cost is the deviation of key measurement from the desired measurement. Consider the uninterrupted LPV system

$$\dot{x} = A(\rho(k))x_k + B(\rho(k))u_k \qquad (18)$$

$A(\rho(k))$ and $B(\rho(k))$ can be stabilized. State feed back control, $u_k = Fx_k$ to stabilize the system based on the following two specifications

1. Transient response specification
2. Magnitude constraints on x_k and u_k

Controller setting:

Choose $Q: R^s \to R^{n\times n}$ and $R: R^s \to R^{n\times m}$ such that $Q = MM^T$ with $(A(\rho(k)), M)$ detectable and $R = R^T > 0$.

Solving the algebraic raccati equation

$$PA(\rho(k)) + A(\rho(k))^T P + Q - PB(\rho(k))R^{-1} B(\rho(k))^T P = 0$$

The above equation is solved for P, which is used in calculation of feedback gain matrix $F: R^s \to R^{n\times 1}$ as

$$F = -R^{-1}B(\rho(k))^T p \qquad (19)$$

States are feed back to the system with gains computed in feed back gain matrix *F*. Simulating the initial response of

$$\dot{x} = (A(\rho(k)) + B(\rho(k)F)x \qquad (20)$$

For different initial conditions of the transient response specification and the magnitude constraints, the response of the system is checked. Typically Q and R are taken as

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & \cdots & 0 \\ 0 & q_{22} & 0 & \cdots & 0 \\ 0 & 0 & q_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & q_{nn} \end{bmatrix} \qquad (21)$$

$$R = \begin{bmatrix} r_{11} & 0 & 0 & \cdots & 0 \\ 0 & r_{22} & 0 & \cdots & 0 \\ 0 & 0 & r_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & r_{nm} \end{bmatrix} \qquad (22)$$

Order of Q depends on order of $A(\rho(k))$ and order of R depends on order of $B(\rho(k))$.

VI. Numerical Simulation Results

Boeing 747 series 100/200 aircraft has been taken as a model for simulation purpose in this paper. This case study has been chosen since it has wide array of characteristics (leading and trailing edge flaps, spoilers, variety of control surfaces, four fan jet engines) which makes of it the perfect representative for any of the commercial airplanes flying today. For B747-100/200 aircraft, the system dynamics has got the coefficients as function of parameter (ρ) [1]. The parameter (α), and (v_{tas}) have got specific bounds.

(α) varies in $[-2, 8]^o$

(v_{tas}) varies [150 250]m/s.

A. LPV Model of Boeing 747

The standard model for the above mentioned aircraft is given by

$$\begin{bmatrix} \dot{x}_k \\ y_k \end{bmatrix} = \begin{bmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix} \begin{bmatrix} x_k \\ u_k \end{bmatrix} \qquad (23)$$

Where the system dynamics $A(\rho) \to R^{n\times n}$, $B(\rho) \to R^{n\times m}$ $C(\rho) \to R^{p\times n}$ and $D(\rho) \to R^{n\times q}$ with affine parameter dependence given by

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$$\left.\begin{aligned} A(\rho) &= A_0 + \sum_{i=1}^{7} (A_i \rho_i) \\ B(\rho) &= B_0 + \sum_{i=1}^{7} (B_i \rho_i) \\ C(\rho) &= [0 \quad I_3] \\ D(\rho) &= 0 \end{aligned}\right\} \qquad (24)$$

An input (u_k) has been taken as sinusoidal and initial states's value $x(0)$ has been taken zero. $D(\rho)$ has taken zero for simulation purpose. The parameter (ρ) is given by

$$[\rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4 \quad \rho_5 \quad \rho_6 \quad \rho_7] = \left[\bar{\alpha} \quad \bar{v}_{tas} \quad \bar{v}_{tas}\bar{\alpha} \quad \bar{v}^2_{tas} \quad \bar{v}^2_{tas}\bar{\alpha} \quad \bar{v}^3_{tas} \quad \bar{v}^4_{tas}\right]$$

Where

$$\left.\begin{aligned} \bar{\alpha} &= \alpha - \alpha_{|trim|} \\ \bar{v}_{tas} &= v_{tas} - v_{tas|trim|} \end{aligned}\right\} \qquad (25)$$

Trim values of the system's states 0are

$$\left[\alpha_{|trim|} \quad q_{|trim|} \quad v_{tas|trim|} \quad \theta_{|trim|}\right] = \left[1.05^0 \quad 0^0/s \quad 227.02\, m/s \quad 1.05^0\right]$$

Trim values of the system's inputs are

$$\left[\delta_{e|trim|} \quad \delta_{s|trim|} \quad T_{n|trim|}\right] = \left[0.163^0 \quad 0.590^0 \quad 42{,}291\, N\right] \qquad (26)$$

B) Simulation Results

This section describes all the simulation results associated with Boeing 747 series 100/200. The LPV system described in the previous section is observed with and without LQR controller. Outputs of the system actual, measured and estimated (with LPV Kalman filter) are shown.

1) LPV System without Controller: The system is observed for unit step input. The step response is shown in figure.

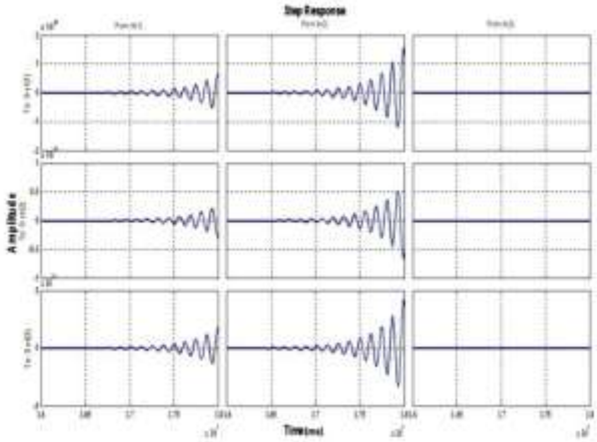

Fig. 1. Step Response without Controller

It can be seen from the that the system is unstable. The output of the system is unbounded for the bounded step input. So to stabilize the system LQR controller is implemented.

2) Performance of system with Controller: Implementing LQR controller the system is observed for step input. The result is shown in the figure.

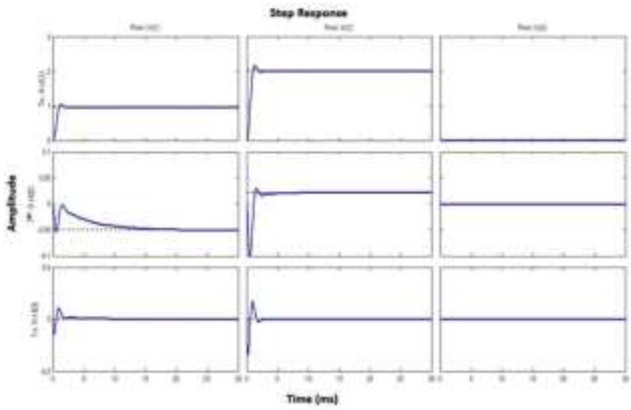

Fig. 2. Step Response with Controller

The figure shows that applying LQR controller the system is stabilized. The output is bounded for bounded step input. For this stable system LPV Kalman filter is implemented.

***3) Output of LPV Kalman filter**:* LPV Kalman Filter is implemented for the above mentioned numerical example. The estimated states with LPV Kalman filter are

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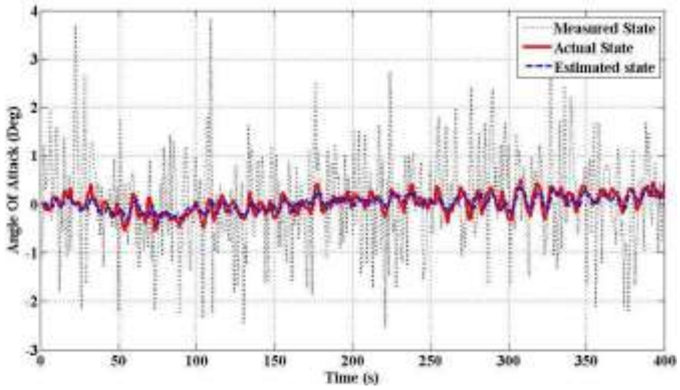

Fig. 3. Angle of Attack

measured angle of attack and estimated angle of attack. It shows that the measured angle of attack show much more deviation from the actual angle of attack, while the estimated angle of attack is much closer to the actual angle of attack.The output of LPV Kalman Filter (estimated angle of attack) is much more better than the measured state.

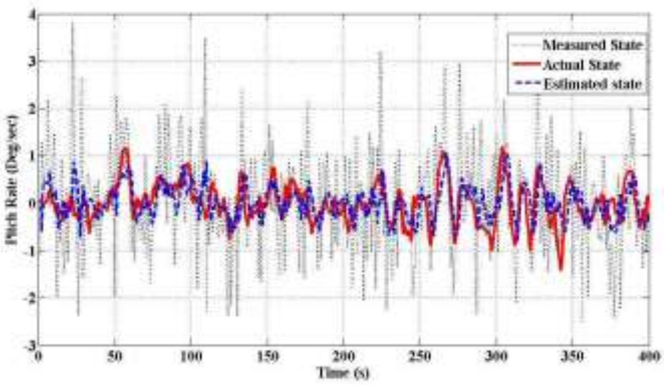

Fig. 4. Ptch Rate

The figure shows pitch rate. Solid line represents actual pitch rate, dotted line represents measured pitch rate and dashed line represents the estimated pitch rate. It shows that the measured pitch rate show much more deviation from the actual pitch rate, while the estimated pitch rate is much closer to the actual pitch rate.The output of LPV Kalman Filter (estimated pitch rate) is much more better than the measured pitch rate.

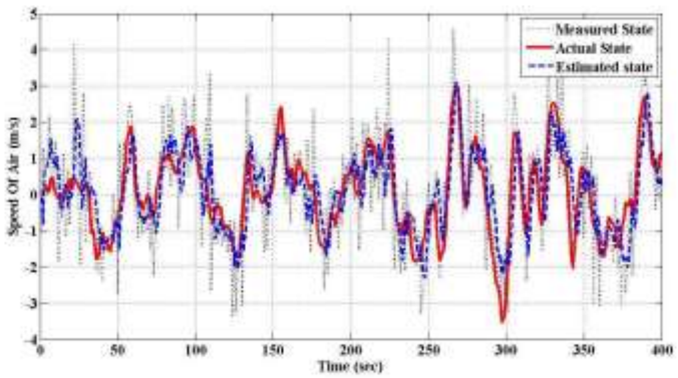

Fig. 5. Speed

The figure shows speed. Solid line represents actual speed, dotted line represents measured speed and dashed line represents the estimated speed. It shows that the measured speed show much more deviation from the actual speed, while the estimated speed is much closer to the actual speed.The output of LPV Kalman Filter (estimated speed) is much more better than the measured speed.

SUMMARY

In this paper, necessary modifications that appears in the conventional Kalman filter when applied to a Linear Parameter Varying systems are studied. Basic steps involved in designing of Kalman filtering are derived for LPV systems. These modifications are observed for case study of Boeing 747 series 100/200 LPV model. For controlling purpose a standard state feedback controller namely LQR is employed. Simulation results obtained are distinguished for attractive features of LPV system.

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