



# Adaptive Inverse Filter Design for Linear Minimum Phase Systems

H. Ahmad, W. Shah

**Abstract**—Adaptive Inverse Filter (AIF) is the standard, tracking, control technique or method which has provided an extensive range of uses and applications for the last several years. This research paper deals with the adaptive inverse Filter (AIF) structure which is being utilized for the stabilized or stable linear systems. Also closed loop features of the AIF are similar as that of the low pass adaptive filtering. Hence, it reduces the consequences of disturbance and the noise. The simulation outcomes for the Linear Minimum phase system or plants are presented to validate the worth of the proposed scheme. AIF has displayed enhanced results in terms of the tracking output.

**Keywords**— The Adaptive Tracking, Least Mean Square (LMS), Linear Minimum Phase System.

## I. INTRODUCTION

This Control systems may be classified as self-correcting type and non-self-correcting type. The term self-correcting, as used here, refers to the ability of a system to monitor or measure a variable of interest and correct it automatically without the intervention of a human whenever the variable is outside acceptable limits. Adaptive inverse filter is a Control method in which the filter must adapt itself to the system parameters which vary, or are initially uncertain. For example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption; a control law is needed that adapts itself to such changing conditions.

Adaptive control is different from robust control in that it does not need a priori information about the bounds on these uncertain or time-varying parameters; Robust control guarantees that if the changes are within given bounds the control law need not be changed, where as in adaptive control the control law changes (adapts) during the operation of the system.

Most filtering algorithms are based on the feedback mechanism. The idea of adaptive filtering is different from this. The filter is in series with the plant, but there is no direct feedback from the plant output to the control input. However, the method is not strictly feed forward (open-loop) control since the controller parameters are being adapted at all times with information from the output of the plant and the command input. Many schemes have been developed for control of linear

minimum phase plants. The problem of achieving perfect tracking of a given reference signal for minimum phase zeros in system inversion procedure is discussed in [1]. Steering along Zeros Control (SAZC) technique for achieving a stable inversion of discrete-time minimum phase linear systems is presented. Diophantine equation is used for obtaining feed forward control unit. This paper discusses the stable inversion of Single Input Single Output (SISO), discrete time, minimum phase linear systems in SAZC terms. Perfect tracking of desired output is achieved with a bounded control action for SISO plants. This technique can also be used for control of Multi Input Multi Output (MIMO) systems. A low order robust (LOR) stable and proper controller for linear SISO systems is given in [2]. Feedback from output of the plant is used to achieve good tracking performance. An unstable hydraulic piston controlled by a servo-valve is considered as a control problem. This paper allows tackling the tracking control problem for SISO systems, guaranteeing robust performance and robust stability for stable and unstable plants. Another scheme shows the problem of robust tracking of non-minimum phase systems under a sensitivity constraint [3].

The schemes discussed above are non-adaptive. For linear time-invariant (LTI) processes these schemes work well but shows poor performance when the environment becomes unknown or slowly time varying. To overcome deficiencies of conventional schemes, adaptive schemes perform robust tracking and show computationally less expensive characteristics. Adaptive Inverse Control (AIC) is a popular tracking method due its wide range of applications for several years [4], [5]. Several techniques used for adaptive control are developed for control of linear or non-linear stable/stabilized plants. These include, pole/zero placement algorithm, simple self-tuning control algorithm, multi-step model algorithm, adaptive PID control with non-parametric identification, gain scheduling and model reference adaptive systems (MRAS) [6], [7]. Adaptive control is treated as a three part problem in [8]. First the plant is stabilized, then plant dynamics are controlled via feed forward control and finally, plant disturbance is filtered by using disturbance canceler. Furthermore, feed forward control and disturbance canceler is used for optimal control of Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) linear and nonlinear plants. An indirect scheme for adaptive control of dynamics of Hammerstein type nonlinear plant using NLMS adaptive filter (FIR) is presented in [9]. The scheme works for minimum and non-minimum phase plants. The FIR filter is designed as an L-delay approximate inverse of the given plant. Simulation results are presented for noise of different characteristics shows nice tracking of plant dynamics. Most of the techniques used for AIC are indirect. In indirect AIC the right inverse of the

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plant is estimated first and then it is used as a left inverse (the controller). Practically, the left inverse may not be equal to the right inverse of the plant as it may comprise of some non-linearity of the plant output. Adaptive filter has been proposed in [10] which mitigates this drawback of indirect AIC. Adaptive filter is applicable to both minimum phase and non-minimum phase plants. The concept of adaptive inverse control is very central to adaptive filtering [11],[12].follow.

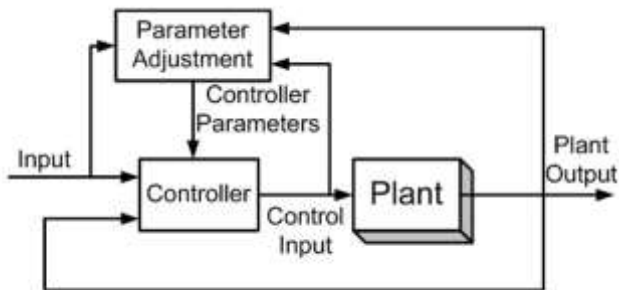


Fig. 1. Adaptive Filter Block Diagram

An approach to adaptive-inverse filter is depicted in Figure 1 to develop simple, robust, and precise filter. The elementary idea of adaptive-inverse filter is to drive the system/plant with a controller signal whose transfer-function is the inverse of that of the system/plant itself. Control input is synthesized to cause the plant output, follow the command-input. Feed forward control is used to monitor the adjustable parameters of the filter. Usually, control of adaptable parameters is done by an algorithm (adaptive) called least square algorithms which reduce the means square of the error to approximate the desired output. Generally, least mean square (LMS) and Normalized Least Square (NLMS) procedures are used for weight updating by minimizing the plant modification error between plant output and estimated output of the algorithm.

## II. LINEAR ADAPTIVE FILTERING

The concept of adaptive filtering is basics to adaptive inverse control. Adaptive filter is manipulated as a building block in many systems. Adaptive filters can be joined with additional structure blocks to make adaptive inverse control schemes [11], [13]. The form of adaptive filter shown in Figure 2, includes a tapped interruption line, adjustable weights (adjustable gains) and an adder to add the weighted signals. This filter is also known as Transversal filter or Tapped delay line filter or FIR filter  $x(k)$  is the input.  $y(k)$  is the output of the filter  $w_0, w_1$  and so on represents weights (or parameters).  $Y(k)$  is the weighted sum of input  $x(k)$  and its delayed versions  $x(k - 1), x(k - 2), \dots, x(k - n)$ . Numbers of delay elements utilizes determines the finite duration of its impulse response.

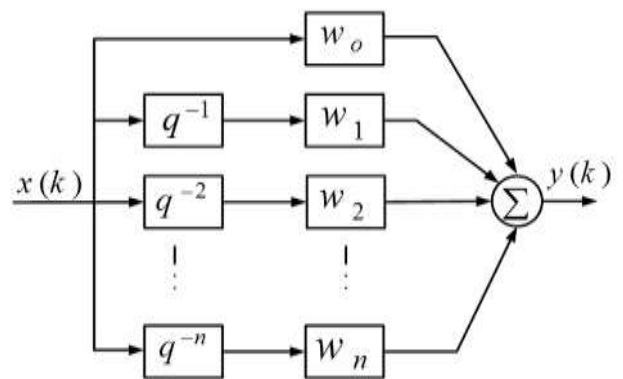


Fig. 2. Adaptive Filter

The output can be written as

$$y(k) = x(k)w_0 + x(k - 1)w_1 + \dots + x(k - n)w_n \quad (1)$$

$$y(k) = \sum_{i=0}^n x(k - i)w_i \quad (2)$$

Equation1 is known as finite convolution sum. Linear output signal  $y(k)$  can also be written by inner product of augmented vectors, shown in Equation3.

$$y(k) = X(k)W^T(k) = W(k)X^T(k) \quad (3)$$

Where

$$X(k) = [x(k) \quad x(k - 1) \quad \dots \quad x(k - n)] \quad (4)$$

$$W(k) = [w_0 \quad w_1 \quad \dots \quad w_n] \quad (5)$$

## III. LEAST MEAN SQUARE ALGORITHM

The LMS weight updation technique can be established by conventional gradient descent method. Weight updation vector  $W(k)$  can be obtained by minimizing the cost or quadratic function synthesized in Equation6 for a given input and desired data patterns [14].

$$\xi(k) = E(e^2(k)) \quad (6)$$

Either non-iterative or iterative algorithms can be used for computing weight vector [15]. Without loss of simplification it can be anticipated that input and desired pattern are statistically stationary.

Where as,

$$e^2(k) = (d(k) - y(k))^2 \quad (7)$$

where,  $e(k)$  is error at instant  $k$  between desired output  $d(k)$  and output of linear combiner  $y(k)$

$$e^2(k) = d^2(k) - 2d(k)X(k)W^T(k) + W(k)X^T(k)X(k)W^T(k) \quad (8)$$

Taking Expectation on both sides

$$\xi(k) = E[e^2(k)] = E[d^2(k)] - 2E[d(k)X(k)]W^T(k) + W(k)E[X^T(k)X(k)]W^T(k) \quad (9)$$

The term in the Equation 9 is called Mean Square Error.

#### IV. SIMULATION RESULTS

After A Linear Minimum phase system is given below

$$y(k) = q^{-\frac{N(q^-)}{D(q^-)}} u(k) \quad (10)$$

$$D(k) = 1 + 0.5q^{-1} + 0.6q^{-2} \quad (11)$$

$$N(k) = 1q^{-1} + 1.4q^{-2} \quad (12)$$

In this example, we choose  $\mu_1 = 0.01$  and  $\mu_2 = 0.03$ .  $\delta$  is being selected with a value of 0.2. Correspondingly the learning rate is being selected 0.03 for AIF. Also the Sampling time selected is .001 sec. The simulations are shown in the Figure 3 to 7.

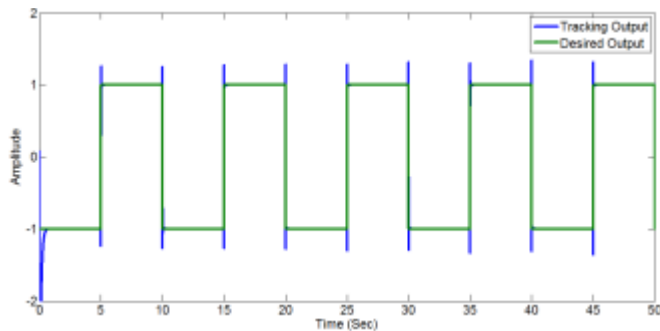


Fig. 3. Output Tracking of Linear Minimum Phase System

The desired or wanted response which is  $y(k)$  and the tracking for the desired or wanted output is being presented in the Figure 3 and Figure 4. The Plant output in the AIF has a low peak value of overshoot and also have less oscillations and furthermore it converges to the wanted or desired output rapidly. MSE for AIF is shown in Figure 5 and Figure 6. MSE is less for AIF. Tracking error is shown in Figure 7. The amplitude of tracking error is less and it also converges to value of zero quicker in the AIF.

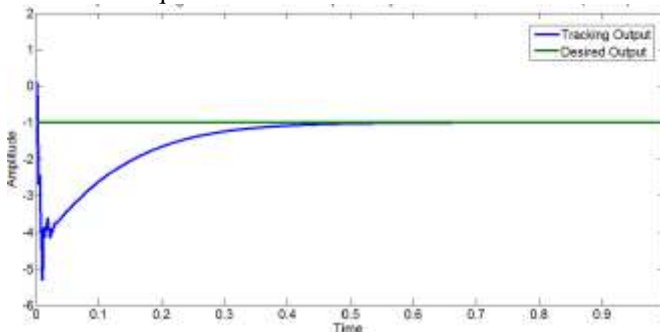


Fig. 4. Output Tracking of Linear Minimum Phase System (1 Sec)

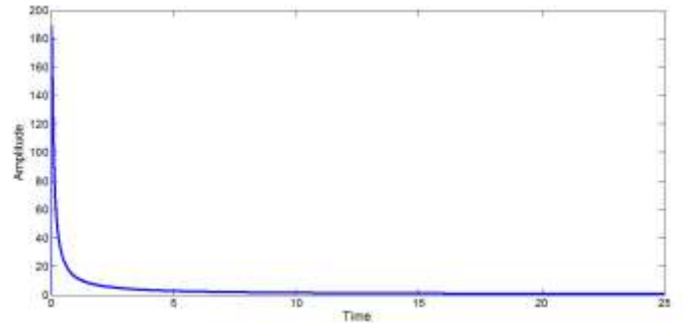


Fig. 5. Mean Square Error for Linear Minimum Phase System

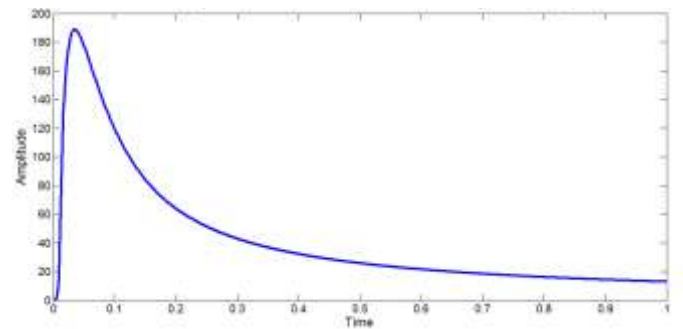


Fig. 6. Mean Square Error for Linear Minimum Phase System (1 Sec)

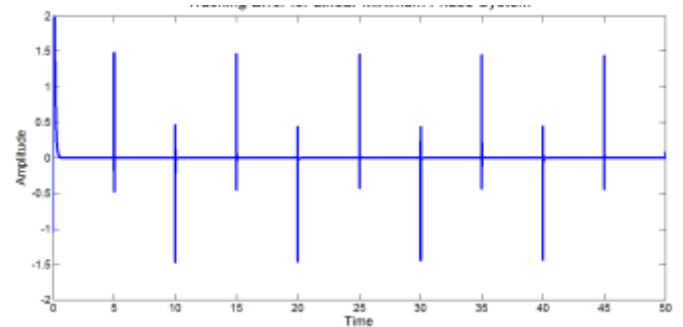


Fig. 7. Tracking Error for Linear Minimum Phase System

#### CONCLUSION

Adaptive Inverse Filter (AIF) based on linear adaptive filtering is appropriate equally to minimum phase discrete time systems. Direct feedback from the system is used to accomplish adaptive tracking. The close loop characteristics are similar to a low pass adaptive filter. Due to this, AIF becomes less sensitive to noise and disturbance hence performs better adaptive tracking. Simulation results show that AIF in terms of tracking desired signal.

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